Semester 1 (Unit 3) Examination, 2020

Question/Answer Booklet

MATHEMATICS METHODS

Section Two: Calculator-assumed

Student Name/Number: ____

Teacher Name: ____

Time allowed for this section

Reading time before commencing work: ten minutes

Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor: This Question/Answer Booklet
Formula Sheet

To be provided by the candidate:

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	12	12	100	100	65
					100

Instructions to candidates

1. The rules for the conduct of School exams are detailed in the School/College assessment policy. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **(100 Marks)**

This section has **12** questions. Answer **all** questions. Write your answers in the spaces provided. Spare pages are included at the end of this booklet.

Suggested working time: **100 minutes**.

Question 8 **(5 marks)**

A certain type of chemical compound contains radioactive isotopes of a particular chemical element. The concentration $c\%$ of each isotope t years from now is given by a formula of the form

$$c = Ae^{-kt}, \text{ where } A \text{ and } k \text{ are constants.}$$

The half-life of isotope X is 36.5 years, and its present concentration in the compound is 0.03%. The half-life of isotope B is 62.9 years, and its present concentration is 0.02%.

(a) Show that for isotope X, $A = 0.03$ and $k = 0.019$ (to 3 decimal places). (3 marks)

(b) When will the concentration of isotope B be twice that of isotope X? (2 marks)

Question 9 **(9 marks)**

A group of medical researchers is investigating the spread of the H2C2 virus in a province by keeping track of P, the population of people infected, t weeks after the virus is first discovered.

Initially, 50 people are infected with the virus. It is suspected that P and t are related by the differential equation

$$\frac{dP}{dt} = -5e^{-\frac{t}{5}} + K \qquad \text{where } K \text{ is a constant.}$$

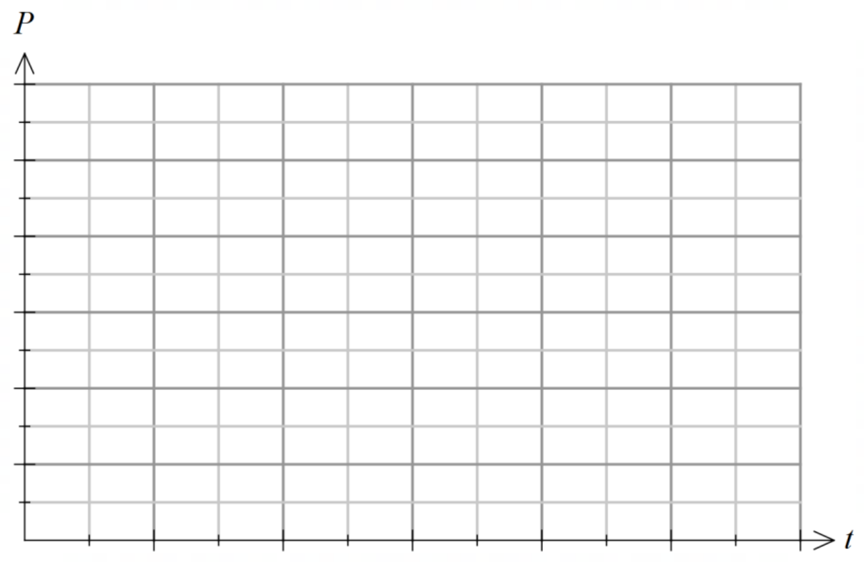
(a) Determine an expression for the population of people infected, P, in terms of t and K. (2 marks)

(b) What happens to the population of infected people when:

(i) $K = 1$? (1 mark)

(ii) $K = 0$? (1 mark)

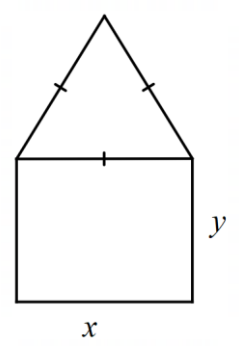
(iii) On the axes provided below draw a sketch to illustrate the two cases considered. ie when $K = 1$ and when $K = 0$. (2 marks)



(c) Given that one week later the population of infected people reduces to half of its initial number, predict what will eventually happen to the population of infected people? (3 marks)

Question 10 **(6 marks)**

A window consists of an equilateral triangle sitting on top of a rectangle, as shown in the diagram below. The sides of the rectangle are x metres and y metres. It is also known that the perimeter of the window is 10 metres.



(a) Show that the area of the window, A is given by

$$A = \frac{(\sqrt{3}-6)}{4}x^2 + 5x$$

(3 marks)

(b) Determine the maximum possible area of the window. (3 marks)

Question 11 **(8 marks)**

The probability distribution of the discrete random variable X is shown in the table below. The expected value of X is 3.5.

x	0	1	3	4	a
$P(X=x)$	$\frac{1}{10}$	b	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$

(a) Determine the value of a and b. (4 marks)

(b) Calculate

(i) the standard deviation of X. (3 marks)

(ii) the standard deviation of $3-2X$ (1 mark)

Question 12 **(5 marks)**

(a) Use your calculator to complete the following table. (2 marks)

h	a	$\dfrac{a^h - 1}{h}$
0.1	2	
0.01	2	
0.001	2	
0.0001	2	

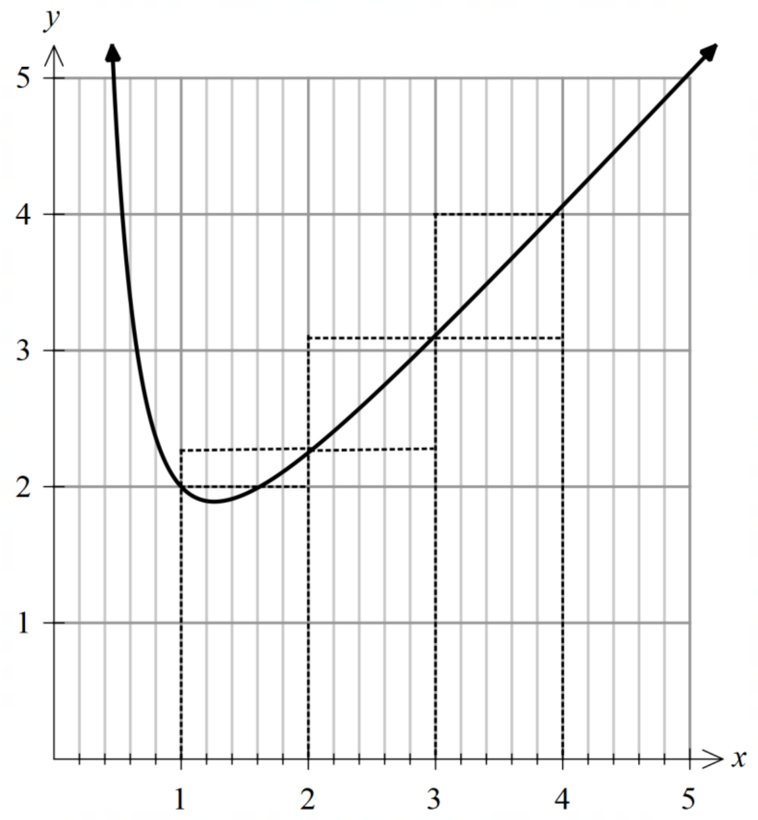
(b) Estimate $\lim\limits_{h \to 0} \dfrac{2^h - 1}{h}$ correct to 3 decimal places. (1 mark)

(c) (i) Use your calculator to determine the number, a which has the property $\lim\limits_{h \to 0} \dfrac{a^h - 1}{h} = 3$ (1 mark)

(ii) State the number a such that $\lim\limits_{h \to 0} \dfrac{a^h - 1}{h} = 1$. (1 mark)

Question 13 **(7 marks)**

The area of the region between the curve $y = x + \frac{1}{x^2}$ and the x-axis for $1 \le x \le 4$ is estimated using three rectangles of equal width of 1 unit. Six rectangles of width 1 unit are shown in the diagram below.



(a) Complete the table of values below expressing all answers as mixed numerals.

(1 mark)

x	1	2	3	4
$y = x + \frac{1}{x^2}$	2			

(b) By considering the rectangles given

(i) show that $\frac{265}{36}$ can be obtained as a lower estimate for the area under the curve from $x=1$ to $x=4$. (2 marks)

(ii) determine an upper estimate for the area under the curve from $x=1$ to $x=4$ in the form $\frac{p}{q}$ where p and q are integers. (2 marks)

(c) Use your answers from part (b) to obtain a better estimate for the area under the curve from $x=1$ to $x=4$. (1 mark)

(d) Determine the exact area under the curve $y=x+\frac{1}{x^2}$ from $x=1$ to $x=4$. (1 mark)

Question 14 **(14 marks)**

The readership of print newspapers has seen a rapid decline in recent years. In one survey researchers collected data across Australia to analyse the readership of both print and digital media.

The data below shows readership results across 7 days for the biggest Australian newspaper publications (Mastheads). The data were published in 2018 and represent Australians aged over 14 years of age.

Cross-Platform Audience (Print & Online)			
All Publications (Mastheads)	**Print Readership**	**Digital Readership**	**Total Readership**
	'000s	'000s	'000s
Australian Total	6547	14 378	18 622

(a) In the table above explain why the entries in the Total Readership column are not the sum of the entries in the Print Readership and Digital Readership columns. (1 mark)

The following Australian data were also obtained for 2018.

Estimated resident population 2018	
Age (years)	Population
0 -14	4 692 750
Over 14	20 289 938
Total	24 982 688

The researchers reported that according to this data,

"32% of all Australians aged over 14 read print media in the 7 day period."

(b) Show how the researchers obtained the value 32%. (2 marks)

A random sample of 10 Australians over 14 years of age was taken.

Let X denote the number of people in this sample who read print media in the 7 day period.

(c) State the probability distribution of X and the mean and variance of this distribution. (3 marks)

(d) What is the probability, correct to four decimal places that

(i) exactly 5 people read print media in the 7 day period? (1 mark)

(ii) there were more people that did read print media in the 7 day period than did not? (2 marks)

(iii) the fourth person selected was just the second of the group to have read print media? (2 marks)

Trial groups of 10 people from each of twenty randomly selected regions were selected.

(e) Determine the probability that within the twenty different regions less than 75% did not read print media. (3 marks)

Question 15 **(4 marks)**

(a) Determine $\frac{d}{dx}\left[\int_0^x f(t)dt + \int_1^x t^3 f(t)dt\right]$. (2 marks)

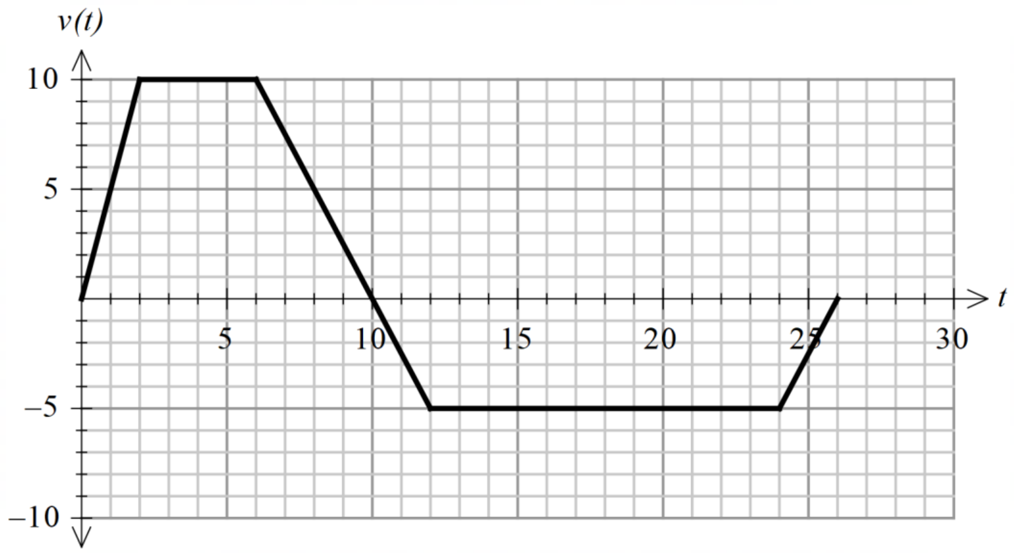
(b) Hence or otherwise find a function *f(x)* such that

$$\int_0^x f(t)dt + \int_1^x t^3 f(t)dt = x^3 + \frac{1}{2}x^6.$$

(2 marks)

Question 16 **(5 marks)**

The velocity, v metres per second, of a particle as a function of time, t seconds, is shown below. The function contains the points, (2,10), (6,10), (10,0), (12,-5), (24,-5) and (26,0).



(a) What is the acceleration when t = 4 seconds? (1 mark)

(b) Determine the distance travelled in the first 6 seconds? (1 mark)

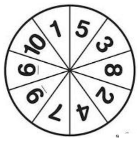
(c) State the displacement of the particle after 12 seconds. (1 mark)

(d) Determine the distance travelled in the first 12 seconds. (1 mark)

(e) At $t = 11$ is the particle slowing down or speeding up? (1 mark)

Question 17 **(14 marks)**

An unbiased spinner with 10 segments numbered from 1 to 10 is spun once. The random variable Y is recorded as a 1 if the result of the spin is a prime number and as a 0 otherwise.



(a) Complete the probability distribution for Y shown below. (2 marks)

y	0	1
$P(Y=y)$		

(b) State the distribution of Y and its mean. (2 marks)

The game "Prime Time" is played at a fair and requires a contestant to spin the above spinner twice. The random variable X is defined as the number of prime numbers obtained in the two spins.

(c) State the possible values X can take. (1 mark)

(d) Determine the probability that $X = 0$. (1 mark)

(e) What is the most likely outcome for X? Justify your answer. (3 marks)

To play “Prime Time”, contestants pay $5.00 to spin the spinner twice. If the player spins 0 or 2 prime numbers he wins $7.00. Otherwise he loses his money.

(f) In one day 500 contestants each play the game once. Calculate the expected gain or loss the game operator is likely to incur on this day. Explain your answer. (3 marks)

Over time the operator noticed that on a daily basis less people were playing the game. He decided to make an adjustment to the price he charged for a contestant to play.

A game is considered “*fair*” if in the long term neither the operator nor the contestant is expected to make a gain or loss.

(g) What price would the operator need to charge to play this game in order to make it “*fair*” without altering the rules or payouts? (2 marks)

Question 18 **(12 marks)**

The amount of daily sunlight (S) falling on a particular place on the Earth's surface can be modelled by a sinusoidal function of the form:

$$S = a + b\cos(ct + d),$$

where S is the number of hours of daily sunlight t months after the beginning of the year, and a, b, c and d are constants.

Given that the daily amount of sunlight ranges from 9.5 hours through to 14.5 hours, and the maximum daily amount occurs 11.7 months after the beginning of the year,

(a) (i) explain why $a = 12, b = 2.5$ and $c \approx 0.5236$. (3 marks)

(ii) use calculus to determine the value of d. (3 marks)

(b) Estimate the amount of sunlight that can be expected on April 30th. (1 mark)

(c) Calculate the average daily amount of sunlight in May and June. (2 marks)

Note: the average value of a function f in an interval $a \le t \le b$ is given by

$$f_{Ave} = \frac{1}{b-a}\int_a^b f(t)\,dt$$

(d) Estimate using the increments formula the biggest change in consecutive days in the total daily sunlight. Give your answer correct to the nearest minute. (3 marks)

Note: for this purpose, you may assume that 1 day is $\frac{1}{30}th$ of a month.

Question 19 **(11 marks)**

Consider the functions $f(x)=\dfrac{2}{1-\sin\left(\dfrac{x}{2}\right)}$ and $g(x)=-(x-2)^2+6$.

(a) Let $A(m)=\int_0^m f(x)\,dx$. Determine the smallest value of m, where m is a positive real number such that $A(m)$ is undefined. (3 marks)

(b) Determine, correct to two decimal places, the total area of the regions bounded by $f(x)$, $g(x)$ and the lines $x=0$ and $x=2$. (5 marks)

(c) Determine c, where $c>0$, if the total area of the regions bounded by $f(x)$, $g(x)$ and the lines $x=0$ and $x=c$ is one half the area of the regions described in (b) above. (3 marks)

Supplementary page

Question number : ____________

Supplementary page

Question number : ____________

Supplementary page

Question number : ____________

Acknowledgements

Question 11

Data source: Roy Morgan
Over 15.7 million Australians red newspapers in print or online
Retrieved 5th March 2020
http://www.roymorgan.com/findings/7878-australian-newspaper-print-

Data source: Australian Bureau of Statistics
Australian Demographic Statistics, June 2019
Retrieved 5th March 2020
https://www.abs.gov.au/AUSSTATS/abs@.nsf/DetailsPage/3235.02018?OpenDocument

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